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## On the Power-Law Distributions of X-ray Fluxes of Solar Flares Observed with the GOES \*

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**Abstract** Power-law frequency distributions of the peak flux of solar flare X-ray emission have been studied extensively and attributed to a system of self-organized criticality (SOC). In this paper, we first show that, so long as the shape of the normalized light curve is not correlated with the peak flux, the flux histogram of solar flares also follows a power-law distribution with the same spectral index as the power-law frequency distribution of the peak flux, which may partially explain why power-law distributions are ubiquitous in the Universe. We then show that the spectral indexes of the histograms of soft X-ray fluxes observed by GOES satellites in two different energy channels are different: the higher energy channel has a harder distribution than the lower energy channel, which challenges the universal power-law distribution predicted by SOC models and implies a very soft distribution of thermal energy content of plasmas probed by the GOES. The temperature ( $T$ ) distribution, on the other hand, approaches a power-law distribution with an index of 2 for high values of  $T$ . Application of SOC models to statistical properties of solar flares needs to be revisited.

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**Key words:** Sun: flares — Sun: X-rays, Gamma rays — Methods: statistical

## 1 INTRODUCTION

Power-law frequency distributions exist ubiquitously in nature, e.g., the magnitude of earthquakes (Gutenberg & Richter 1954), the frequency that a word is used in literature (Zipf 1949). More frequency distributions following a power law can be found in Clauset et al. (2009) and Aschwanden (2011). For a power-law frequency distribution, the number of events  $dN$  scales with the magnitude of the event  $x(> 0)$  as a power-law function:

$$dN = Ax^{-\delta}dx, \quad (1)$$

where the coefficient  $A > 0$  and the power-law index  $\delta$  are constant. Usually the distribution deviates from a power-law function towards the low end of the magnitude  $x$ . This deviation can be attributed either to the breakdown of the power-law scaling or to some observational bias (Li et al. 2013).

There have been quite a number of statistical works on the X-ray emission from solar flares. The X-ray peak flux has a power-law frequency distribution with a power-law index varying from 1.6 to 2.1 for different studies (e.g., Hudson et al. 1969; Drake 1971; Shimizu 1995; Lee et al. 1995; Feldman et al. 1997; Shimojo & Shibata 1999; Veronig et al. 2002; Yashiro et al. 2006; Aschwanden & Freeland 2012). Without a background subtraction, Veronig et al. (2002) found that the peak soft X-ray (SXR) flux of flares obey a power-law distribution over three orders of magnitude from the flare GOES class C2.0 to X20. Feldman et al. (1997) divided flares observed by the GOES into different groups according to the background level and used the background-subtracted peak flux of flares for statistics. They found that the power law distribution can be extended down to A1.0 class flares. Based on the aforementioned statistical studies, Aschwanden & Freeland (2012) summarized in their Table 2 the total number of flares observed by the GOES, the flux range where the frequency distribution is consistent with a power-law, and the corresponding power-law indices. They found that these observations can be explained with a fractal-diffusive avalanche model (Aschwanden 2012; Du 2015).

Although the peak flux of large flares can be easily obtained due to their high values, the value of the peak flux for small flares is always contaminated by the background emission, instrumental noise, and potential flare identification bias. The latter may be overcome by using the histogram of the flare flux, which incorporates

properties of the light curve with the peak flux distribution, to study the statistics. [Zhang & Liu \(2015\)](#) recently showed that the characteristics of the soft X-ray light curve do not vary with the value of the peak flux. The histogram of the flare flux is therefore intimately related to the peak flux distribution.

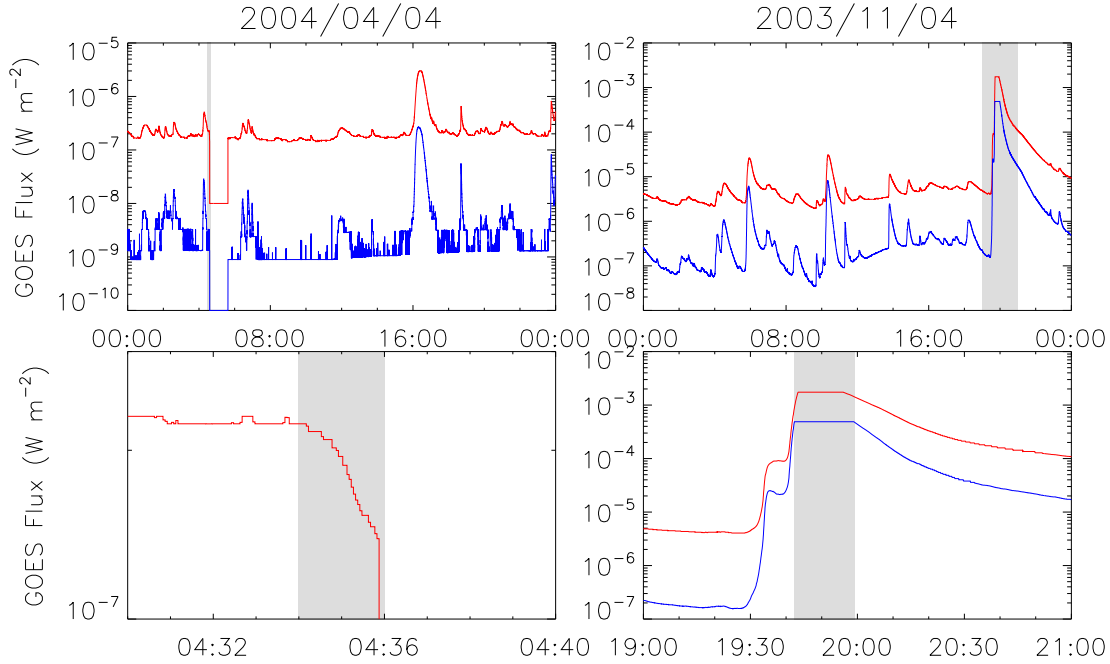
A series of GOES satellites have taken a huge amount of SXR flux measurement of the Sun over 40 years. In this paper, the data obtained from 1981 to 2012 are used. Moreover to reduce the effect of the selection bias, instead of identifying individual flares, we will include all data points at an original GOES time cadence of 3 s before 2009 and 2 s after 2009 to study the statistics of the differential histograms of the GOES fluxes, which are different from but intimately connected to the frequency distribution of the GOES peak fluxes studied before. The GOES data reduction is presented in Section 2. The histograms are shown in Section 3. In Section 4, we explore the origin of the power law distribution of the differential histogram and its deviation from a power law towards the low value end of the flux. Conclusions and discussions are presented in Section 5.

## 2 GOES DATA REDUCTION

Since 1974, a series of GOES satellites have been put into operation and measured continuously the total soft X-ray emission flux at two wavelengths: 1-8 and 0.5-4 . The time cadence was 3 s before December 1 2009 and has been improved to 2 s afterwards. More information on GOES data can be found in [Aschwanden & Freeland \(2012\)](#) and references therein. High temporal-resolution GOES data from 1981 to 2012 are used in this study.

Before carrying out detailed investigation, one needs to treat caveats in the obtained data properly. Two types of anomaly in the GOES data are illustrated in Figure 1. In the left panels, the sudden drop of the flux from 04:36 to 05:38 UT is due to the entry of the satellite into the shadow of the Earth. To remove the effect of such data on the histogram of the flux, we identify local minimums of the flux associated with the earth occultation and ignore 150 data points before and 60 data points after these minimums.

The right panels show the other type of anomaly due to instrumental saturation ([Ryan et al. 2012](#)). The saturated data are marked by shaded regions from 19:42 to 19:59 UT. This type of anomaly only affects the histogram at extremely-high flux values, which can be readily identified in the histogram. In total, there are more than 322 million data points in our sample.

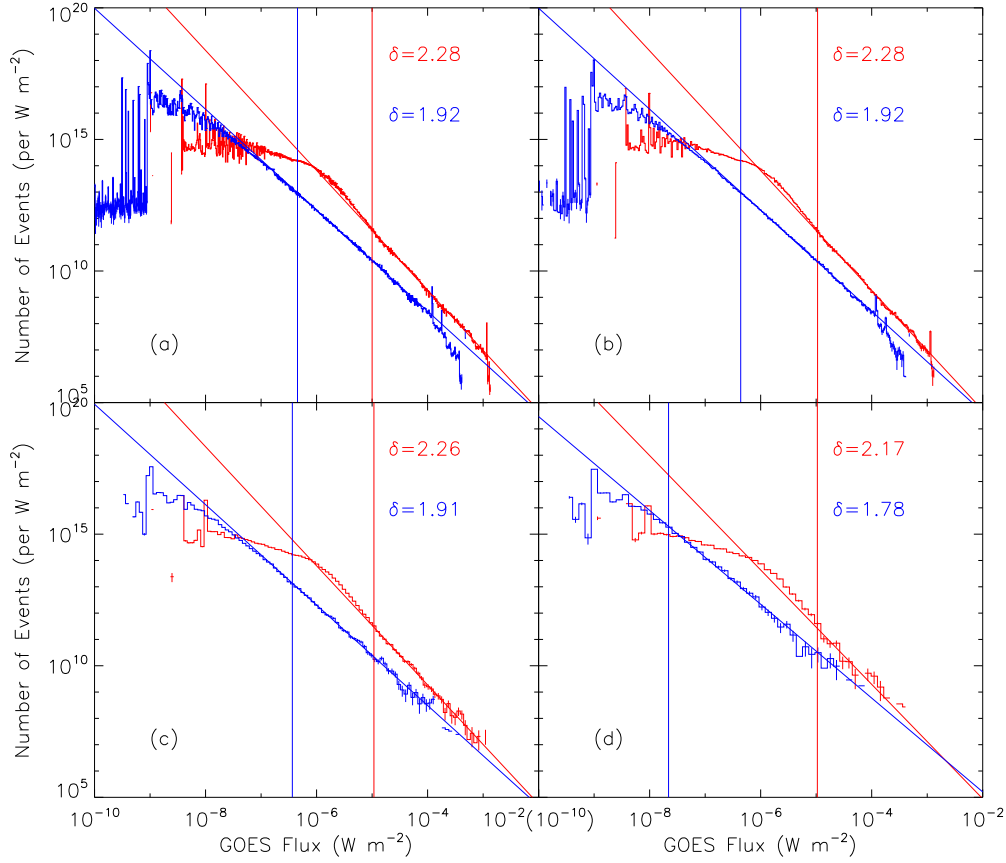


**Fig. 1** Two types of anomalies in the GOES data . Left panels: sudden drop of flux due to the Earth occultation; Right panels: instrumental flux saturation.

### 3 OBSERVATIONAL RESULTS

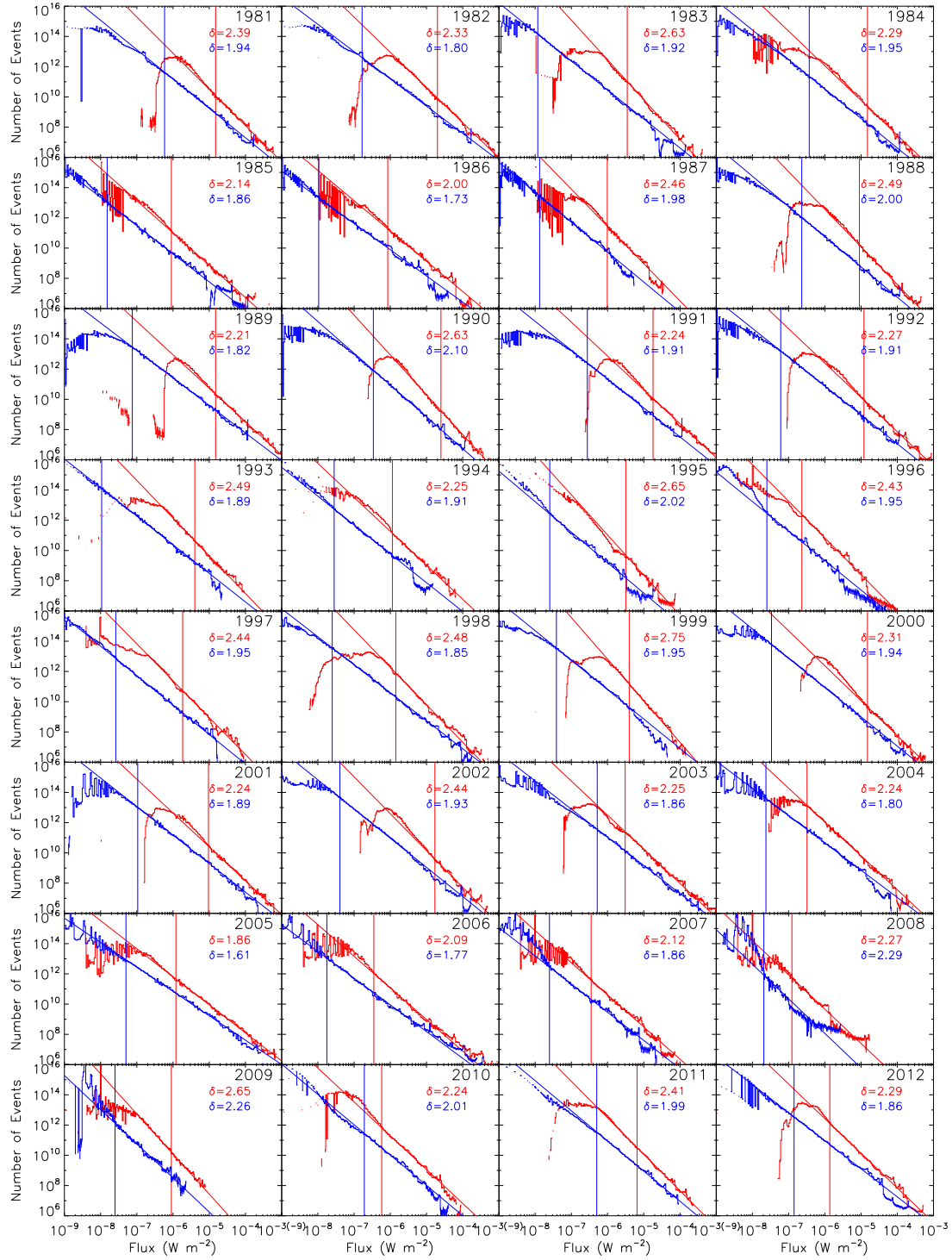
Panel (a) of Figure 2 shows the differential histograms of these fluxes in two energy channels. It is evident that the differential histograms of the fluxes follow a power law distribution towards the high value end of these fluxes. With the maximal-likelihood fitting procedure (Crosby et al. 1993; Clauset et al. 2009; Li et al. 2012), we fitted each histogram with a power law model. The power-law index for the lower energy band is bigger than that of the higher energy channel. The spikes at the high value end of the fluxes are caused by the flux saturation mentioned above and have been excluded in our fitting. Panel (b-d) in Figure 2 are the histograms of a subset of the sample obtained with different sampling cadence, which have the same distribution as the one of complete sample.

To investigate the variation of the histograms in the solar cycle, different panels of Figure 3 show the histograms of the flux observed in different years. Although the power law indexes show significant variation with the time, the index for the low energy band is always bigger than that for the high energy band. And except in 2005 dur-



**Fig. 2** Differential histograms of the original data and those with coarser samplings. The red color is for the 1 - 8 Å GOES flux, and the blue one for the 0.5 - 4 Å flux. The fittings of these histograms with a power law model (Clauset et al. 2009) are indicated with straight lines with the indexes given in the corresponding figures. The vertical lines mark the lower cutoffs of the corresponding power laws. Panel (a): histograms of the full sample. (b) histograms of the data sampled with a cadence of 1-minute/40-s (for a time resolution of 3s and 2s, respectively); (c) histograms of the data sampled with a cadence of 1-hour/40-minute; (d) histograms of the data sampled with a cadence of 1 day/16 hours.

ing the solar minimum, the index in the low energy band is always greater than 2, which is consistent with the frequency distribution of the peak flux (Aschwanden & Freeland 2012).



**Fig. 3** The histograms and their corresponding fittings in each year from 1981 to 2012. The year is indicated in the upper-right corner in each panel. The colors and symbols have the same meaning as those in Figure 2.

#### 4 INTERPRETATION OF THESE HISTOGRAMS

Soft X-ray emission observed by the GOES is mostly produced via thermal bremsstrahlung process with the flux density given by:

$$F(e) \propto EM T^{-1/2} \exp(-e/k_B T), \quad (2)$$

where  $T$ ,  $EM$ , and  $e$  represent the plasma temperature, emission measure, and photon energy, respectively and  $k_B$  is the Boltzmann constant. The observed power-law distribution of soft X-ray fluxes in different energy bands can be used to derive the frequency distribution of  $T$  and  $EM$ . From  $F(e_l)^{-\delta_l} dF(e_l) \propto F(e_h)^{-\delta_h} dF(e_h)$ , where  $e_l$ ,  $e_h$ , and  $\delta_l$ ,  $\delta_h$  represent the high- and low-energy bands and the corresponding power-law indexes of their histograms, respectively, we have

$$[EM T^{-1/2} \exp(-e_l/k_B T)]^{1-\delta_l} \propto [EM T^{-1/2} \exp(-e_h/k_B T)]^{1-\delta_h}, \quad (3)$$

$$EM \propto T^{1/2} \exp\{[(\delta_l - 1)e_l - (\delta_h - 1)e_h]/[k_B T(\delta_l - \delta_h)]\}, \quad (4)$$

$$F(e_l) \propto \exp\{[(\delta_h - 1)(e_l - e_h)]/[k_B T(\delta_l - \delta_h)]\}, \quad (5)$$

$$F(e_h) \propto \exp\{[(\delta_l - 1)(e_l - e_h)]/[k_B T(\delta_l - \delta_h)]\}, \quad (6)$$

and the frequency distribution of  $T$  is then given by:

$$D(T) \propto T^{-2} \exp\{[(\delta_l - 1)(\delta_h - 1)(e_h - e_l)]/[k_B T(\delta_l - \delta_h)]\}. \quad (7)$$

The frequency distribution of  $EM T^{-1/2}$  follows a power-law with an index of  $\alpha = [\delta_l(\delta_h - 1)e_h - \delta_h(\delta_l - 1)e_l]/[(\delta_h - 1)e_h - (\delta_l - 1)e_l] \geq \delta_l$ . **If  $\delta_l = \delta_h$ , then  $\alpha = \delta_l$ , and equation (2) implies that  $T$  needs to be a constant. Therefore different spectral indexes for the histograms of low and high channels are intimately related to the temperature distribution and the correlation between  $T$  and  $EM$ . Equations (5) and (6) show that the harder distribution of  $F(e_h)$  is caused by the greater dependence of  $F(e_h)$  on  $T$  than  $F(e_l)$ .**

It is interesting to note that the temperature distribution  $D(T)$  approaches a power-law distribution with an index of 2 at high values of  $T$ . The fast increase of the distribution toward low values of  $T$  may be attributed to contribution from the background plasma no necessarily associated with individual flare events. Then in SOC models, one should associate the quantity with a universal power-law distribution with an index of 2 with intensive variable  $T$  instead of fluxes or thermal energy, which are combinations of intensive and extensive variables.

Equation (7) shows that  $1/T$  follows an exponential distribution with a cutoff of  $[k_B(\delta_l - \delta_h)]/(\delta_l - 1)(\delta_h - 1)(e_h - e_l)]$ . The temperature of hot plasmas detected by the GOES therefore distributes in a relatively narrow range, which is consistent with results given by [Ryan et al. \(2012\)](#). Assuming that volume  $V$  of the emission region is not correlated with  $T$  and density  $n$ , which appears to be the case for existing solar flare observations ([Li et al. 2012](#)), the thermal energy of the emitting plasma is then proportional to  $nVT \propto (EMV)^{1/2}T$ . Considering the narrow distribution of  $T$ , the thermal energy of the emitting plasma therefore follows a power distribution with an index of  $2\alpha - 1 > 3$ , where we have used the fact that  $\alpha > \delta_l > 2$ . Therefore hot plasmas with a low energy content dominates the thermal energy associated with flares ([Hudson 1991](#)).

One should emphasize that the histograms of GOES fluxes studied here are different from frequency distributions of the flare peak flux commonly investigated. However, if the time evolution of the flux is assumed to be independent of the peak flux:

$$F = F_p f(t),$$

where  $0 < f(t) \leq 1.0$  and  $f(0) = 1$  describe the statistically averaged time evolution of the normalized flare X-ray flux and  $F_p$  is the peak flux at the peak time  $t = 0$ , the frequency distribution of the observed flux of a given flare is then given by:

$$\frac{dn(F)}{dF} \equiv \left| \frac{dt}{T dF} \right| = \frac{f}{T F f'},$$

where  $T$  is the sampling interval,  $f' = df/dt$ .

We note that light curve of the normalized flux may vary drastically from one flare to another. However, as shown by [Zhang & Liu \(2015\)](#), the statistical properties of flare light curves indeed does not vary with  $F_p$ . One may divide light curve of the normalized flux of all flares into several groups depending on their level of similarity with each group represented by a characteristic light curve. The  $f(t)$  is then the weighted mean of these characteristic light curves. If  $F_p$  follows a power-law distribution with an index of  $\delta$  above some cutoff frequency, the frequency distribution of  $F$  for all flares is then given by:

$$\frac{dN(F)}{dF} \propto \int_F^\infty F_p^{-\delta} dF_p \frac{dn(F)}{dF} = F^{-\delta} \int_1^\infty f(t)^{\delta+1} \frac{dt(f)}{T df} d\left(\frac{1}{f}\right),$$

which has the same spectral index as the distribution of  $F_p$ .



**The frequency distribution of the peak flux of hard X-ray emission also follows a power-law distribution. To extrapolate the results above to higher energies, one may assume that**

$$F(e) \propto \exp\{[(\delta_l - 1)(e_l - e)]/[k_B T(\delta_l - \delta)]\}, \quad (8)$$

**where  $\delta$  is the index of the flux distribution at  $e$ . Then we have**

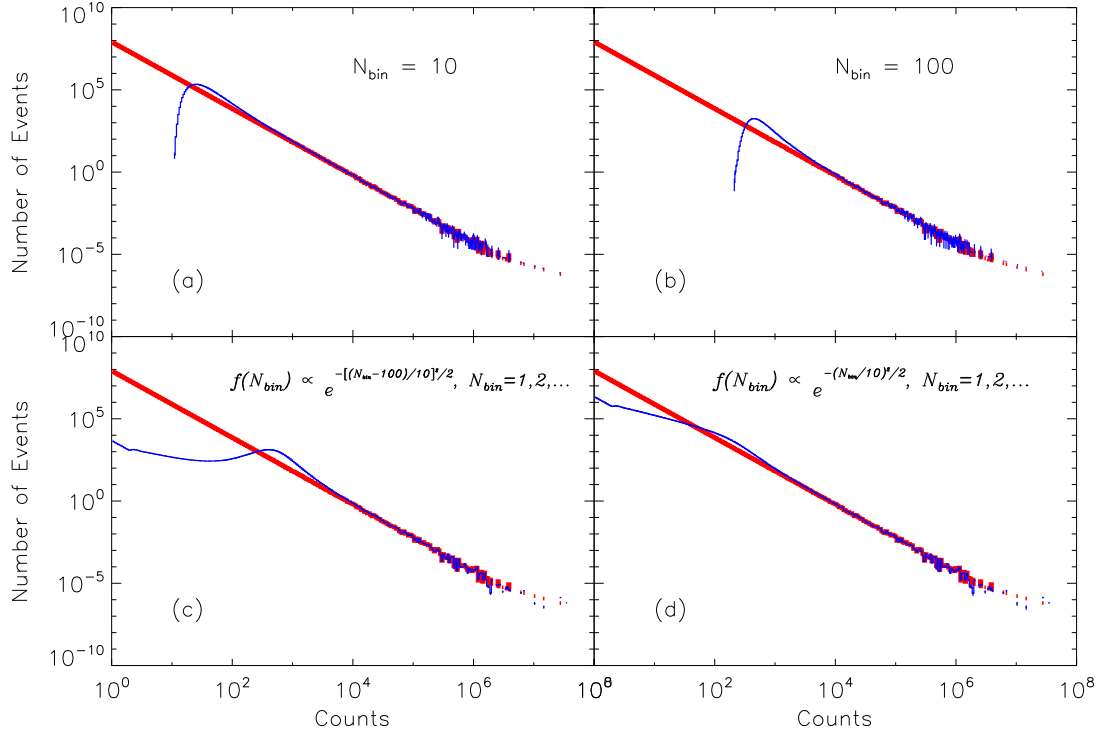
$$\delta = \frac{(\delta_l - \delta_h)(e - e_l) + (\delta_h - 1)(e_h - e_l)\delta_l}{(\delta_l - \delta_h)(e - e_l) + (\delta_h - 1)(e_h - e_l)}, \quad (9)$$

**which can be tested with future observations. Deviation from such a correlation between  $\delta$  and  $e$  will invalidate the isothermal model (2) for the flux.**

The histograms studied above deviate from a power-law distribution toward the low value end of the flux. At a given time, GOES records the soft X-ray flux coming from all features occurring in the solar disk. In fact when we check a full-disk image in SXR, e.g., an image observed by the Soft X-ray Telescope onboard YOHKOH (Tsuneta et al. 1991), we can see active regions, sometimes with flares superposing on it (e.g., Li et al. 2012), and bright points (e.g., Shimojo & Shibata 1999; Zhang et al. 2001), etc. Therefore, each single datum we had in the sections above is actually the superposition of the flux from a number of elementary phenomena. For simplicity, we call an elementary phenomenon an element from now on.

If we assume the frequency distribution of the flux/count from these elements follows the same power law, it would be interesting to know how the frequency distribution produced by the superposition of a number of elements looks like. We started from an original power-law distribution with an index of  $\delta = 2.03$ , with a total number of 80 million elements, and with a lower cutoff of 1. This distribution is shown in Figure 4 by a thick red line. To obtain the blue curve in panel (a), we randomly selected 10 data points along the original power law in red, and summed their counts to form a new data point. We repeated this process for  $8 \times 10^7 / N_{\text{bin}}$  times, and obtained  $8 \times 10^7 / N_{\text{bin}}$  samples. The frequency distribution of the counts of this new samples is indicated by the blue curve in Figure 4 (a). One immediate finding is that after superposing 10 elements at each data point, a “bump” appears in the range about 10 to 100 counts. And the drop from the power law distribution at the lower end is very rapid. If we increase the value of  $N_{\text{bin}}$  to 100 as indicated in panel (b), the deviation from the power law occurs below a higher value at about  $5 \times 10^3$  counts, and the “bump” also shifts rightwards.

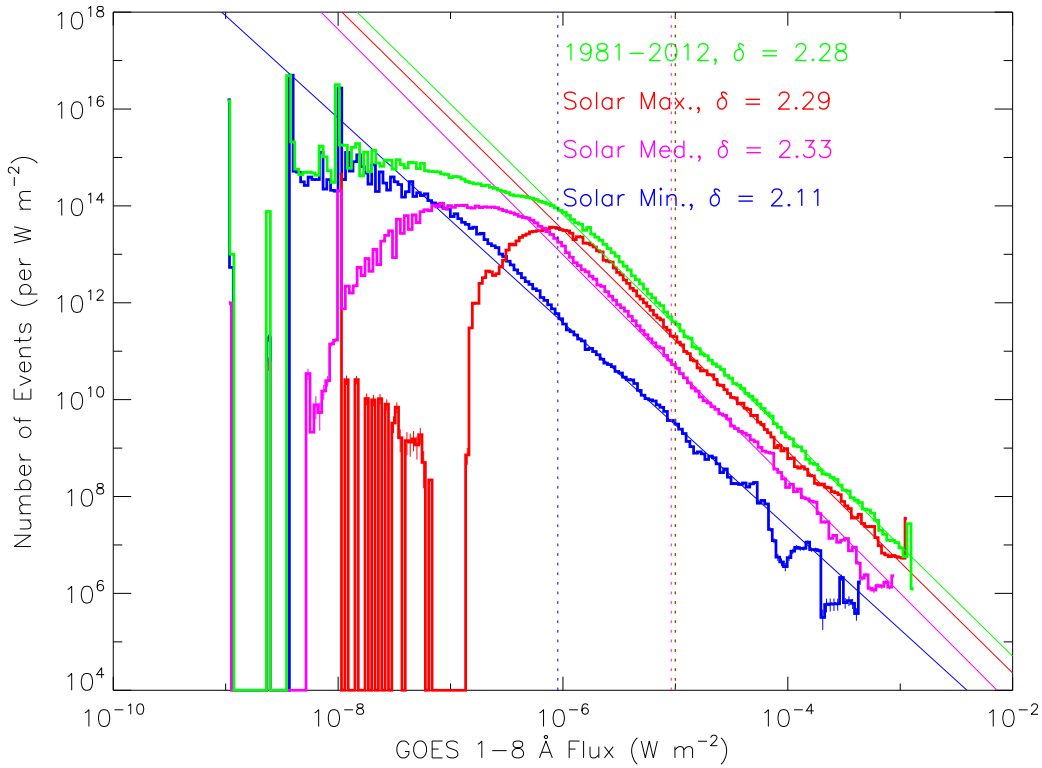
As the number of elements on the solar disk is not constant with time, we may assume that  $N_{\text{bin}}$  follows a Gaussian distribution. In panels (c) and (d), we adopt two



**Fig. 4** A demonstration of the element superposition model. The original frequency distribution of the counts of elements is indicated in red in each panel. Panel (a): randomly select  $N_{bin} = 10$  elements, and sum up their counts to form a new event point. The blue curve is the frequency distribution of the counts of such defined new event. Panel (b): Similar to the distribution in panel (a), but  $N_{bin} = 100$ . Panel (c) and (d):  $N_{bin}$  follows a gaussian distribution indicated at the top of the panel.

Gaussian distributions as indicated in the upper region of both panels. When comparing the deviation part of the distribution from the power law between constant and non-constant  $N_{bin}$  cases, we find the “bump” regions in panels (c) and (d) are less prominent, and do not have a rapid drop toward the low end as the cases in panels (a) and (b). Interestingly, such distributions in panels (c) and (d) have similar shape to the distributions shown in Figure 2.

If we attribute the “bump” feature in the histogram to the superposition of elementary sources, we can expect that the histogram varies with solar activities. At solar activity maximum, there are more elementary sources on the disk to be superposed for



**Fig. 5** Histograms of GOES 1 - 8 Å flux at different levels of solar activity. The curve in red shows the distribution of the flux measured around the solar activity maximum. The curve in blue shows the result around the activity minimum. The one in magenta is the result derived from the flux at medium activity level. The green one is the distribution of the flux measured over the entire period from 1981 to 2012. The power-law indices of the fittings are indicated in the upper right corner, and vertical lines mark the flux lower cutoffs.

a measured GOES flux. While at solar activity minimum, the number of superposing elements are much lower.

To classify the 32 years from 1981 to 2012 according to the level of solar activity, we used the flare occurring rate provided by [Aschwanden & Freeland \(2012\)](#) as a criterion. Seven years had the flare occurring rate below 3000 per year are classified to the solar minimum group. They are 1985, 1986, 1995, 1996, 2007, 2008, and 2009. Six years had the flare rate above 15000 per year belong to the solar maximum group.

They are 1989, 1990, 1991, 2000, 2001, 2002. The solar medium activity group has the flare rate between 8000 to 13000 per year. We found that 1983, 1992, 1993, 1998, 2004, 2011 could be included in this group.

In Figure 5, the histograms of GOES 1 - 8 Å flux at different levels of solar activity are illustrated. The curves in red, blue, and magenta represent the histograms of the flux measured around the activity maximum, minimum, and medium, respectively. The green curve is the histogram of the flux measured over the entire period. As a first step to compare the results among the periods of maximum, medium, and minimum activity, we again applied the method in Clauset et al. (2009) to fit a power law to each curve. The best-fit power-law indices are presented in the upper right corner. The vertical lines mark the corresponding lower cutoffs of the power laws. It also indicates the position below which a “bump” appears. We also find that the “bump” at solar maximum is in a flux range about one order of magnitude larger the “bump” region at solar minimum.

A more sophisticated fitting method is to use the element superposition model. The fitting parameters include the power-law index  $\delta$ , the Gaussian center and width which describes the distribution of the superposition number  $N_{bin}$ , a ratio to convert counts into GOES flux, and another parameter to convert the model frequency to the frequency of GOES flux. Therefore, there are five free parameters. In Figure 6 the four histograms of GOES 1 - 8 Å flux during the period of minimum, medium, maximum activity, and the entire 32 years are fitted with the element superposition model using non-linear least square method. The best-fit power-law index and Gaussian parameters are marked in the upper right corner.

The black solid lines are the best-fit power laws for the elementary events. They can be extended down to the flux indicated by the vertical lines, which could be regarded as the corresponding lower cutoffs. We can see that the lower cutoffs vary with the level of solar activity. It implies that the distribution of these elementary events also varies with the level of solar activity. The fittings in Figure 5 have the power-law indices of 2.11, 2.33, 2.29, and 2.28 for the period of solar activity minimum, medium, maximum, and the entire 32 years, whereas here the fittings of the superposition models in Figure 6 produce harder indices of 2.00, 2.09, 2.08, and 2.09, respectively. It is interesting to note that the last three indices are consistent with each other, while the distinction of the first one from the others may be attributed to its inclusion of data from all level of the solar activity. Furthermore, the power-law distributions of these new fittings cover much broader ranges of GOES flux of about five orders of magnitude from about  $10^{-8}$  to  $10^{-3} \text{ Wm}^{-2}$ .

The superposition model can fit the measured histograms fairly well. The thick gray lines in Figure 6 represent the results of the best-fit superposition models and their corresponding fitting ranges. For the GOES flux during the entire 32 years, the superposition model is able to fit the entire range of the histogram as indicated in green color. For the GOES flux during the solar minimum, the model produces higher values than the measured histogram at the fluxes higher than  $7 \times 10^{-5} \text{ W m}^{-2}$ . For the GOES flux during the solar maximum and medium, the results of the best-fit models are delineated by two black dashed lines which are extrapolated down to lower flux values. The models have higher number of events than the measured values close to the lower end. The deviation of the measured histogram at either higher or lower flux end from the model may be due to the selection bias when picking up the events occurring during the period of the maximum, medium, and minimum activity separately. The discrepancy at the lower flux end may also suggest the presence of an X-ray background during the solar active phases, which is distinct from the elementary events proposed above. The histogram in green color takes into account all possible events occurring from 1981 to 2012. Therefore, it suffers the least influence of selection bias and produces the most convincing result.

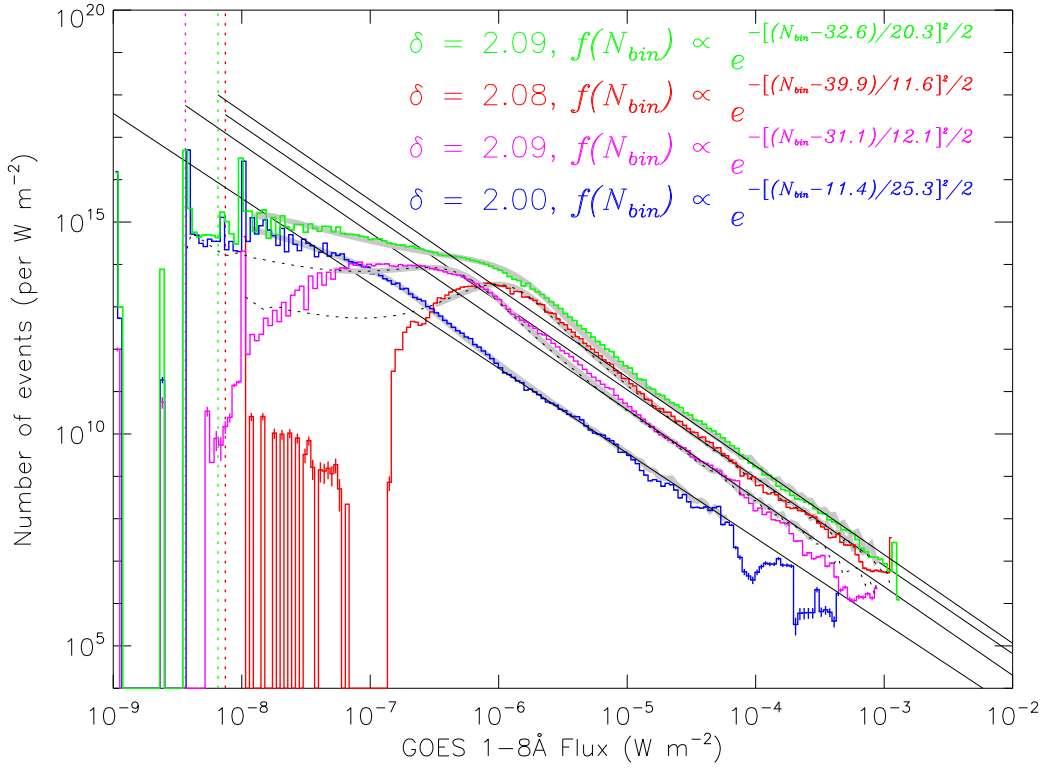
When we compare the Gaussian peak value of  $N_{bin}$  in different activity periods, we find that it is proportional to the level of solar activity. Around solar maximum, the most probable  $N_{bin}$  is about 40. This value is larger than the one of 31.1 during the medium activity period, and much larger than the one of 11.4 during the solar minimum. These results could be understood straight forward, as the number of elements occurring on the solar disk increases with the level of solar activities.

## 5 CONCLUSIONS AND DISCUSSIONS

In this paper we have shown theoretically that if the shape of the flare light curve is not correlated with the peak flux, the differential histogram of the flare flux shares the same power-law distribution as the frequency distribution of the peak flux. Observationally, we have investigated the statistics of all usable GOES 1 - 8 Å and 0.5 - 4 Å flux observed from 1981 to 2012 to minimize the effect of selection bias on frequency distributions. There are two major findings in our work.

(1) The histograms of two GOES channels obey power laws with different indices. The index of the power law for the 0.5 - 4 Å GOES flux is harder than the one for the 1 - 8 Å flux. And these two indices do not change with the sampling cadence.

(2) A “bump”-like structure is clearly seen in all the histograms of the 1 - 8 Å flux. It could be interpreted by the element superposition model proposed in this paper. The



**Fig. 6** Fittings of the element superposition model to the histograms of GOES 1 - 8 Å flux at different levels of solar activity. Colors have the same definition as those in Figure 5. The solid gray lines indicate the results of the best-fit superposition models and their corresponding fitting ranges. The dashed lines represent the best-fit models to the histograms during the maximum and medium activities extrapolated down to lower flux values. The best-fit model parameters are marked in the upper-right region. The power law distributions of the elementary phenomena are shown by the black solid lines with their lower cutoffs marked by the vertical lines.

original element frequency distribution of the entire 32-year data is a power law with an index of 2.09. This index is harder than the one derived from the fitting with the maximum likelihood method. The best-fit parameters of superposed sources  $N_{\text{bin}}$  is correlated with the level of solar activity.

The GOES 1 - 8 Å peak flux of flares without background subtraction has been found to follow a power law with an index greater than 2.1, e.g., [Veronig et al. \(2002\)](#) obtained an index of 2.11, and [Yashiro et al. \(2006\)](#) derived an index of 2.16. However, when the background is subtracted, the 1 - 8 Å peak flux of flares produced a harder index a bit below 2 ([Lee et al. 1995](#); [Feldman et al. 1997](#); [Aschwanden & Charbonneau 2002](#)). This difference between with and without background subtraction can possibly be interpreted by the element superposition model. The case with background subtraction is equivalent to the case with less number of superposed elements, as background subtraction would remove all other elementary sources other than flares. Actually, the frequency distribution of the background-subtracted flare peak flux could be linked to the upper portion of the distribution of the elementary sources.

According to the theory of self-organized criticality (SOC) ([Aschwanden 2012](#)), [Aschwanden & Freeland \(2012\)](#) predicted that the peak flux of flares in SXR has a power-law frequency distribution with an index of 2. From GOES 1 - 8 Å flux measurements, the frequency distributions of background-subtracted flare peak flux during 1975 to 2011 produced a power-law index of  $1.98 \pm 0.11$  ([Aschwanden & Freeland 2012](#)). Our result of the power-law index of 2.09 (see Figure 6) is a little bit softer than the theoretical value. The obtained power-law index in the 0.5 - 4 Å waveband using the maximal-likelihood fitting method (see Figure 2) is about 1.92. The single event in this waveband is also a superposition of a number of elementary sources. If we use the superposition model to derive the original element frequency distribution, we would probably get a lower power-law index than 1.92, which was derived using the maximum likelihood method. As the “bump” structure is not very pronounced in this waveband, we did not use the superposition model for a further fitting. According to the SOC theory, the hard X-ray (HXR) peak flux of flares have a power-law index of 1.67. The emission from 0.5 - 4 Å probably contains some contribution from HXR. Therefore, the theoretical index might be between 1.67 to 2. Our power-law index of  $< 1.92$  is consistent with the theoretical expectation. These results also indicate that the power-law distribution of X-ray fluxes from solar flares involves convolution of complex physical processes over a broad scale range and may not be simply attributed to some scaling indexes of simple mathematical models.

We have to note that the power law index of 2.09 we have derived is the statistical result over 32 years. We have tried to minimize the bias of event selection, as existing in flare statistics ([Parnell & Jupp 2000](#); [Aschwanden & Charbonneau 2002](#); [Li et al. 2013](#)). However, if we go to the distribution in each year as shown in Figure 3, the power-law index for the 1 - 8 Å flux distribution ranges from 1.86 to 2.75, and for the

0.5 - 4 Å flux, it ranges from 1.61 to 2.29. Therefore, the power-index is quite time dependent. In particular, in year 2005 and 2008, the power indices are very different from the values in other years.

In our simple element superposition model, the original power-law distribution has a lower cutoff  $x_0$ . It is not a necessity, other forms of lower-end deficiency can be used, such as saturation. As mentioned in the introduction, 300 samples can cover the flux in two orders of magnitude in the power-law distribution with an index of 2. For our 322 millions of data points, in principle they can cover eight orders of magnitude data. However, in Figure 2 the apparent power law only covers two orders of magnitude. After removing the superposition effect, the frequency distributions of elementary sources could be able to cover five orders of magnitude data.

Due to the instrumental saturation of the very high GOES flux, we can not know the exact upper limit of the flux. By extrapolating the frequency distribution of elementary sources to the flux greater than  $10^{-2} \text{ W m}^{-2}$  (equivalent to a X100 class flare), we find that this extremely high flux may occur 1000 times per 32 years. As our sampling frequency is 1/3 Hz most of time, it corresponds a time period of 3000 s, similar to the lifetime of a large flare. That is to say, we should be able to observe a X100 class flare within 32 years. Have we observed this kind of super flares? We do not know. It may hide in the saturated data.

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## References

- Aschwanden M. J., Charbonneau P., 2002, ApJ, 566, L59 15
- Aschwanden M. J., 2011, Self-organized Criticality in Astrophysics: The Statistics of Nonlinear Processes in the Universe (Berlin: Springer) 2
- Aschwanden M. J., 2012, A&A, 539, 15 2, 15
- Aschwanden M. J. Freeland S. L., 2012, ApJ, 754, 112 2, 3, 5, 11, 15



- Crosby N. B., Aschwanden M. J., Dennis B. R., 1993, *Sol. Phys.*, 143, 275 [4](#)
- Clauset A., Shalizi C. R., Newman M. E. J., 2009, *SIAM Review*, 51, 661 [2](#), [4](#), [5](#), [12](#)
- Drake J. F., 1971, *Sol. Phys.*, 16, 152 [2](#)
- Du, Z. L., 2015, *Ap&SS*, 359:4 [2](#)
- Feldman U., Doschek G. A., Klimchuk J. A., 1997, *ApJ*, 474, 511 [2](#), [15](#)
- Gutenberg B., Richter C. F., 1954, *Seismicity of the Earth and Associated Phenomena* (2nd ed.; Princeton,NJ: Princeton Univ. Press), 310 [2](#)
- Han F.-r., Liu S.-m., 2013, *Chinese Astron. Astrophys.*, 37, 277
- Hudson H. S., Peterson L. E., Schwartz D. A., 1969, *ApJ*, 157, 389 [2](#)
- Hudson, H. S., 1991, *Sol. Phys.*, 133, 35 [8](#)
- Lee T. T., Petrosian V., McTiernan J. M., 1995, *ApJ*, 418, 915 [2](#), [15](#)
- Li Y. P., Gan W. Q., Feng L., 2012, *ApJ*, 747, 133 [4](#), [8](#), [9](#)
- Li Y. P., Gan W. Q., Feng L., Liu S. M., Struminsky, A., 2013, *Research in Astronomy and Astrophysics*, 13, 1482 [2](#), [15](#)
- Parnell C. E., Jupp P. E., 2000, *ApJ*, 529, 554 [15](#)
- Ryan D. F., et al., 2012, *ApJS*, 202, 11 [3](#), [8](#)
- Shimizu T., 1995, *PASJ*, 47, 251 [2](#)
- Shimojo M., Shibata K., 1999, *ApJ*, 516, 934 [2](#), [9](#)
- Tsuneta S. et al., 1991, *Sol. Phys.*, 136, 37 [9](#)
- Veronig A. M., Temmer M., Hanslmeier A., Otruba W., Messerotti M., 2002, *A&A*, 382, 1070 [2](#), [15](#)
- Yashiro S., Akiyama S., Gopalswamy N., Howard R. A., 2006, *ApJ*, 650, L143 [2](#), [15](#)
- Zipf G. K., 1949, *Human Behaviour and the Principle of Least Effort* (Cambridge, MA: Addison-Wesley) [2](#)

Zhang J., Kundu M., White S. M., 2001, Sol. Phys., 88, 337 [9](#)

Zhang P., Liu S. M., 2015, ChAA, 39, 330 [3](#), [8](#)